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STRAIN RATE SENSITIVITY AND YIELD POINT BEHAVIOR IN MILD STEEL

JAMES M. KELLY

Division of Structural Engineering and Structural Mechanics,
Department of Civil Engineering, University of California, Berkeley, California

Abstract—An empirical description of the rate sensitive behavior of mild steel based on a logarithmic dependence of flow stress on strain rate is examined in detail. It is shown that the law provides a good representation of the material response for strains up to about 5·0 percent. The delayed yield phenomenon, which is a striking characteristic of mild steel behavior is included and a mechanism previously found to be accurate in representing the delay time under constant stress is used to determine the delay time under complex stress histories. The relaxation from the upper yield point resulting from the delay time in a constant strain rate situation is prescribed by the rate sensitive behavior of the material.

It is shown that the response of the material is a functional of the stress history and has a fading memory in strain as well as time. The characteristic memory strain is found to be small so that for large strains the influence of the history is small and an equation-of-state representation is appropriate. For small strains and strain histories of rapidly changing type the integral representation must be used.

The theory is applied to the prediction of the response of a beam subject to constant rate of curvature and the predicted response compared with published experimental data. In addition, the transverse impact of a mass on a beam is investigated and the theory is used to predict the onset of yielding in the beam. This problem is of importance in the design of highway guard-rails.

1. INTRODUCTION

THE study of the dynamic plastic behavior of metals is at present a subject of considerable practical importance. A substantial body of experimental work has been published, but not all of it is consistent and there has been much variability in the interpretation of the experimental results. There is, as is well known, some difference in the positions taken by various experimentalists on the question of the rate dependence of certain aluminum alloys. (For the most recent statements on this subject one might refer to the colloquium edited by Huffington [1].) There is no question, however, that iron and mild steel are extremely rate sensitive materials. The amount of experimental data available on strain rate effects in mild steel is not as extensive as that on other metals such as aluminum and copper and this is surprising in view of its commercial importance. It is significant that the flow stress in mild steel has a highly non-linear dependence on strain rate and that the response is further complicated by the presence of a delayed yield phenomenon.

While attempts to describe the rate dependent behavior of plastics and certain other non-metals have been made with varying success within the theories of linear or non-linear viscoelasticity, in metals the approach has been to propose empirical relations between stress, strain and strain rate. Thus interpretations of strain rate effects in metals have been based on an equation of state of the form

$$\dot{\varepsilon}_p = g^*(\sigma, \varepsilon_p)$$

where ε_p is the plastic strain, σ the stress and g^* some suitable function of σ and ε_p. Since

$$\varepsilon_p = \varepsilon - \sigma/E$$

where ε is the total strain and E the Young's modulus this equation may take the form

$$E\dot{\varepsilon} = \dot{\sigma} + g(\sigma, \varepsilon). \tag{1}$$

An equation of this form was introduced by Malvern [2] and attempts have been made to obtain the form of g or g^* experimentally by Rajnak and Hauser [3] for aluminum and by Marsh and Campbell [7] for mild steel.

A relationship such as (1) is not properly an equation of state since by implication an equation-of-state relating three quantities should provide a unique value for one of the three quantities if the other two are specified. Equation (1), even in its simplest form (proposed by Sokolovsky [5] and used by Malvern [2] and others) namely

$$g(\sigma, \varepsilon) = \frac{1}{\tau}[\sigma - f(\varepsilon)], \qquad \tau = \text{const.}, \tag{2}$$

where $\sigma = f(\varepsilon)$ is the "static" stress–strain curve, implies that the stress is a functional (and not a function) of the strain and strain rate; for, writing (1) with (2) in the form

$$\mathrm{d}[\sigma - f(\varepsilon)]/\mathrm{d}t + [\sigma - f(\varepsilon)]/\tau = (E - \mathrm{d}f/\mathrm{d}\varepsilon)\,\mathrm{d}\varepsilon/\mathrm{d}t$$

leads on integration to

$$\sigma = f(\varepsilon) + \int_0^t (E - \mathrm{d}f/\mathrm{d}\varepsilon')\exp\left(-\frac{t-t'}{\tau}\right)\partial\varepsilon'/\partial t'\,\mathrm{d}t', \qquad \varepsilon' = \varepsilon(t')$$

so long as $\varepsilon \geq 0$ and the origin of the time scale is chosen such that $\sigma(0) = 0$ and $\varepsilon(0) = 0$. This indicates that a description which neglects the influence of strain history on stress must be an approximate one.

The representation of g in the form

$$g(\sigma, \varepsilon) = g_1[\sigma - f(\varepsilon)]$$

which is an extension of the simple form used above is indicated by certain experimental results (e.g., on copper by Riparbelli [6]) and can, depending on the form of g_1, lead to a considerable simplification in the mathematical analysis of physical problems involving impact and wave propagation. The primary purpose of the paper is to examine in detail the response of mild steel as predicted by a particular constitutive law of this type and to show that the delayed yield phenomenon can be brought into the theory in a natural way. The relation between stress and strain at constant strain rate is extended to the bending of beams and the resulting moment curvature relation is compared with available experimental data. In addition, the results on the delay time are used to predict the onset of yielding in an impact problem of technical importance.

2. CONSTANT STRAIN RATE BEHAVIOR

2.1 *Strain rate sensitivity*

It is proposed that the response of the material from the unrestrained, unstressed state be described by the two basic equations

$$E\dot{\varepsilon} = \dot{\sigma}; \qquad t \leq t_d \tag{3a}$$

$$E\dot{\varepsilon} = \dot{\sigma}+(\sigma_c/\tau)\{\exp[\sigma-f(\varepsilon)]/\sigma_c-1\}; \qquad t > t_d, \tag{3b}$$

providing $\dot{\varepsilon} \geq 0$.

The quantities σ_c and τ are a characteristic stress and a natural time respectively and the delay time t_d is dependent on the stress history in the interval $0 < t < t_d$. A method for the determination of t_d for specified stress histories will be given in the following section.

In a situation in which the strain is prescribed it is convenient to write (3b) in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\{[\sigma-f(\varepsilon)]/\sigma_c\}+\frac{1}{\tau}\{\exp[\sigma-f(\varepsilon)]/\sigma_c-1\} = (E-\mathrm{d}f/\mathrm{d}\varepsilon)\frac{\mathrm{d}}{\mathrm{d}t}(\varepsilon/\sigma_c) \tag{4}$$

which by suitable manipulations and use of the integrating factor $\exp\{t/\tau+[E\varepsilon-f(\varepsilon)]/\sigma_c\}$ leads to the integral

$$\sigma(t) = f(\varepsilon)-\sigma_c \ln\left\{\exp\left(-\frac{t-t_d}{\tau}\right)\exp\left(-\frac{E\varepsilon-f(\varepsilon)}{\sigma_c}\right)\left(1-\exp\frac{\sigma^*-f(\varepsilon^*)}{\sigma_c}\right)\right.$$
$$\left.+1-\frac{1}{\sigma_c}\int_{t_d}^{t}\exp\left(-\frac{t-t'}{\tau}\right)\exp\left(-\frac{E(\varepsilon-\varepsilon')}{\sigma_c}\right)\exp\left(\frac{f(\varepsilon)-f(\varepsilon')}{\sigma_c}\right)(E-\mathrm{d}f/\mathrm{d}\varepsilon')\partial\varepsilon'/\partial t'\,\mathrm{d}t'\right\} \tag{5}$$

where $\sigma^* = E\varepsilon^*$ is the value of the stress at $t = t_d$ and ε' is used to denote $\varepsilon(t')$.

Since it will be subsequently shown that ε^* is always greater than the strain corresponding to the static lower yield point for all strain rates of practical importance, the fact that $\mathrm{d}f/\mathrm{d}\varepsilon$ is very much less than E over most of the plastic range can be used to simplify the above result. The term $f(\varepsilon)-f(\varepsilon')$ is as a consequence, negligible in comparison to $E\varepsilon-E\varepsilon'$ and the above result reduces to

$$\sigma = f(\varepsilon)-\sigma_c \ln\left\{\exp\left(-\frac{t-t_d}{\tau}\right)\exp\left(-\frac{E(\varepsilon-\varepsilon^*)}{\sigma_c}\right)\left(\exp\left(-\frac{\sigma^*-f(\varepsilon^*)}{\sigma_c}\right)-1\right)\right.$$
$$\left.+1-\frac{E}{\sigma_c}\int_{t_d}^{t}\exp\left(-\frac{t-t'}{\tau}\right)\exp\left(-\frac{E(\varepsilon-\varepsilon')}{\sigma_c}\right)\frac{\partial\varepsilon'}{\partial t'}\mathrm{d}t'\right\}. \tag{6}$$

The representation of stress in terms of strain history thus involves a fading memory in time with a relaxation time τ (using the terminology of viscoelasticity theory) and in addition has a fading memory in strain with the characteristic memory strain given by σ_c/E.

In the case of constant strain rate, $\dot{\varepsilon} = \alpha$ the reduced form of (5) becomes

$$\sigma = f(\varepsilon)-\sigma_c \ln\left\{\exp\left(-\frac{E\varepsilon-f(\varepsilon^*)}{\sigma_c}\right)\exp\left(-\frac{\varepsilon-\varepsilon^*}{\alpha\tau}\right)\right.$$
$$\left.+\left[1-\exp\left(-\frac{\varepsilon-\varepsilon^*}{\alpha\tau}\right)\exp\left(-\frac{E(\varepsilon-\varepsilon^*)}{\sigma_c}\right)\right]\left(1+\frac{E\alpha\tau}{\sigma_c}\right)\right\}. \tag{7}$$

For large strains the above equation may be approximated by

$$\sigma = f(\varepsilon)+\sigma_c \ln(1+E\alpha\tau/\sigma_c). \tag{8}$$

Curves of σ against $\log \alpha$ were given by Marsh and Campbell [7] for mild steel specimens of two grain sizes; one a large grain-size material with 346 grains/mm^2 and the other a small grain-size material having 2030 grains/mm^2. These curves have been replotted for large strains (≥ 2 percent) in Figs. 1(a) and 1(b), as $\sigma - f(\varepsilon)$ against $\log \alpha$. For strain rates exceeding 0·1 sec^{-1} the data lies on straight lines which indicate reasonable values of σ_c and τ to be 26,000 psi and 10^{-2} sec for both materials. It follows that $E\alpha\tau/\sigma_c$ is about 10 when $\alpha = 1$ sec^{-1} and the deviation of the results from a straight line near $\alpha = 0{\cdot}1$ sec^{-1} must then be expected.

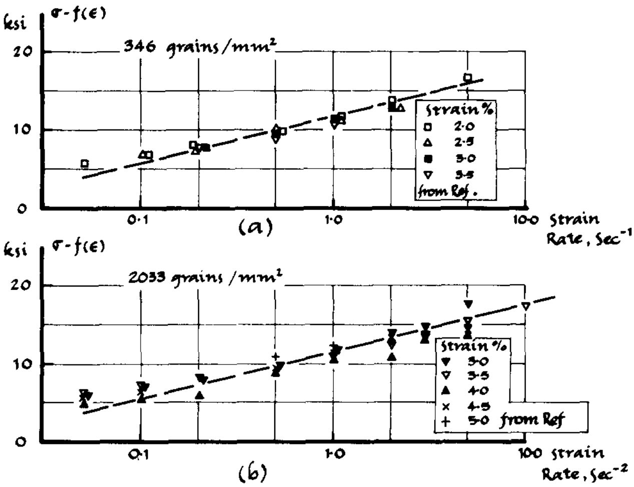

FIG. 1. Stress difference $\sigma - f(\varepsilon)$ against logarithm of strain rate. Replotted from data taken from Marsh and Campbell [7].

The formula above is an asymptotic one and if changes in strain rate have occurred in the strain history of the test from which the data is obtained it is clear from the exact formula (5) that test points corresponding to strains or times close to the changes cannot be given by (7). It is a surprising result that σ_c and τ do not appear, at least from the two materials studied, to be dependent on grain size. This would be an extremely useful conclusion if it were to be substantiated by the data on mild steel of other grain sizes.

2.2 *Yield point determination*

The above formulae contain as a parameter the delay time t_d. To estimate t_d use will be made of published experimental data by Campbell and Marsh [7]. In their tests, mild steel specimens of various grain sizes were loaded dynamically in compression under constant stress. It was found that the delay time is dependent on the applied constant stress and on the grain size. The authors discussed a number of models and found that the one which best corresponds to the data is the hypothesis that macroscopic yielding can only occur when a critical fraction of the grains contain released dislocations. For uniform grain size this gives the result.

$$t_d = \frac{A}{d^3}\left(\frac{\sigma_0}{\sigma}\right)^\beta \tag{9}$$

where σ is the constant stress and σ_0 is twice the shear stress required to release a source in the absence of thermal fluctuations and d is the average grain diameter. Taking σ_0 as $2G/45$ the value of A obtained from the data was $1{\cdot}2 \times 10^{-6}$ mm^3 sec and β was approximately 9.

In situations, as in (6), where the stress is not constant, the determination of the delay time can only be conjectured. It will be assumed here that the above model is valid and that the rate of release of dislocations is a function of the applied stress. Thus, if the critical fraction required for macroscopic yielding is F then the rate R at which this critical fraction is reached under constant stress is

$$R = F \bigg/ \frac{A}{d^3}\left(\frac{\sigma_0}{\sigma}\right)^\beta.$$

Assuming that the same formula will hold for variable stress, the delay time t_d will be given by the solution of

$$\int_0^{t_d} \frac{d^3}{A}\left(\frac{\sigma}{\sigma_0}\right)^\beta \mathrm{d}t = 1. \tag{10}$$

In the case of constant strain rate $\dot{\varepsilon} = \alpha$, this takes the form

$$E\alpha t_d = \sigma^* = \{E(1+\beta)A\sigma_0^\beta \alpha/d^3\}^{1/(1+\beta)}. \tag{11}$$

On this basis the upper yield point σ^* is proportional to 1/10-th power of the strain rate. Since the lower yield point is asymptotically proportional to $\ln \alpha$ the upper yield point will increase more rapidly with increasing α than the lower yield point. Substitution of the values of A, d, β for the two grain sizes gives

$$\sigma^* = 77{,}000\,\alpha^{1/10} \text{ psi for 2030 grains/mm}^2$$

and

$$\sigma^* = 60{,}000\,\alpha^{1/10} \text{ psi for 346 grains/mm}^2.$$

The indicated static yield points for these two materials are 33,000 psi and 26,500 psi respectively and suggest a strain rate of the order of 2×10^{-4} sec^{-1}.

Values of σ^* computed from (11) and of σ_c and τ from the previous section have been inserted in (7) and (8). The resulting stress–strain curves are shown in Fig. 2(a) and 2(b). Superposed on these curves are experimental points taken from Marsh and Campbell [7]. There is considerable scatter which may be explained by the fact that the type of test from which the data is taken was not one in which the strain rate was constant. In fact, the results were obtained from tests in which the stress was approximately constant. The integrated form of the constitutive relations as given in (5) indicates that the stress is a functional of the strain being dependent on the strain history. The exponential terms in the formula show that the material has a fading memory both with respect to time, as for example do linear viscoelastic materials, but also with respect to strain. Thus, if the strain changes

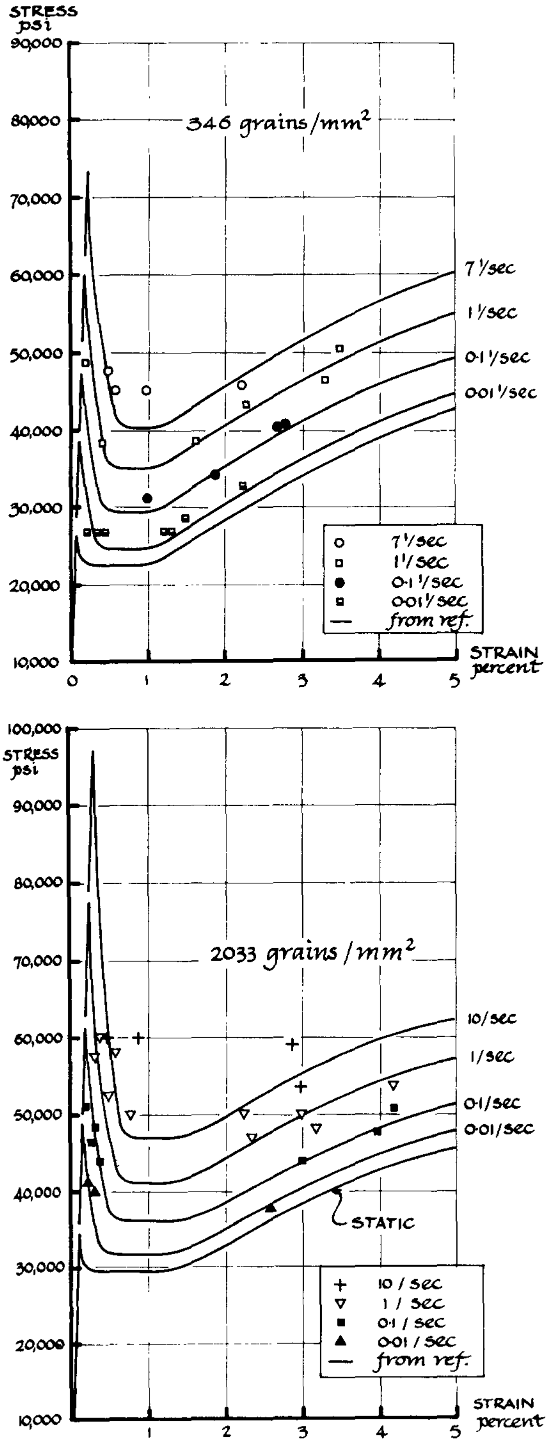

FIG. 2. Stress–strain curves at constant strain rate. Experimental points taken from Marsh and Campbell [7].

by a certain amount which with $\sigma_c = 26{,}000$ psi may be around $\frac{1}{2}$ percent, while the strain rate remains constant the effect of the previous strain history is negligible. Also this can apply only for strains at least $\frac{1}{2}$ percent larger than the strain at which yield takes place since the initial yield is, on the basis of (10), dependent on the stress history up to the point of yielding the material is elastic up to this point. The subsequent relaxation from the yield stress thus cannot be represented by an equation of state approach although the large strain behavior has the possibility of being so represented providing that rapid changes in the strain rate history do not occur. While it is true that the large strain response of materials is of primary interest in understanding the interrelation of physical structure and mechanical response it is the unusual behavior of mild steel at yield and in the immediate post yield regime which is of considerable technical interest and importance in the understanding of impact and related phenomena.

3. APPLICATION TO BEAM BENDING

The preceding development of uniaxial stress–strain relations allows a useful extension to the bending of beams. In obtaining a moment–curvature relation the usual assumptions of beam theory will be made, namely, that plane-sections normal to the middle surface remain plane and normal and that the behavior of the material in tension is identical to that in compression.

Since the strain rate varies linearly through the cross section, the fiber furthest from the neutral axis will have the smallest delay time. Thus the cross section will behave elastically up to a yield moment M^* given by the t_d of the outermost fiber. For times greater than the delay time of the outermost fiber a yield zone will spread into the beam, the stress in fibers lying outside this yield zone will be relaxing from the σ^* corresponding to the strain rate at the fiber in question. The material lying inside the yield zone will have an elastically increasing fiber stress.

A rectangular cross section $(b \times 2c)$ subject to a constant rate of curvature will be considered. The strain at a point in the cross section distant y from the neutral axis is

$$\varepsilon = \varepsilon_{\max} y/c.$$

The maximum elastic moment carried by the section is

$$M^* = \frac{2}{3} bc^2 \sigma^*$$

where

$$\sigma^* = \left(\frac{E(1+\beta) A \sigma_0^\beta \dot{\varepsilon}_{\max}}{d^3} \right)^{1/(1+\beta)}.$$

At any time greater than the delay time of the material at $y = \pm c$ the material in the region $|y| > \bar{y}$ is relaxing viscoplastically. The plastic region is determined by the fact that $\sigma(\bar{y})$ is on the point of yielding. Thus,

$$\begin{aligned} \sigma^*(\bar{y}) &= \left(\frac{E(1+\beta) A \sigma_0^\beta \dot{\varepsilon}_{\max}}{d^3} \right)^{1/(1+\beta)} \left(\frac{\bar{y}}{c} \right)^{1/(1+\beta)}. \\ &= E \varepsilon_{\max} \bar{y}/c \end{aligned}$$

Thus

$$\bar{y}/c = \left(\frac{E(1+\beta)A\sigma_0^\beta \dot{\varepsilon}_{\max}}{d^3}\right)^{1/\beta}(E\varepsilon_{\max})^{-(1+\beta)/\beta}. \tag{12}$$

For $|y| > \bar{y}$ the stress is given by (7) with $\alpha = \dot{\varepsilon}_{\max} y/c$ and in this region the integration with respect to y, needed for the computation of the moment induced in the cross section, must be carried out numerically. For $\varepsilon_{\max}$ greater 0·5 percent the approximation to (7) as given by (8) becomes sufficiently accurate to be used in the computation and the result for the moment can be given in simple terms. The static stress–strain curve, $\sigma = f(\varepsilon)$, for mild steel can usually be represented for strains up to 5 percent by the simple form

$$\begin{aligned} f(\varepsilon) &= E\varepsilon; \qquad \varepsilon < \varepsilon_s \\ &= \sigma^y + F\langle \varepsilon - \varepsilon_w \rangle; \qquad \varepsilon_s < \varepsilon. \end{aligned}$$

In the above $\sigma^y = E\varepsilon_s$ is the static lower yield point, F is a work hardening constant, the term $\langle \varepsilon - \varepsilon_w \rangle$ is taken to be zero if the argument is negative and ε_w is the strain at which work hardening commences. Using this form of stress strain curve the moment is obtained as a function of $\varepsilon_{\max}$ for fixed $\dot{\varepsilon}_{\max}$ as

$$\begin{aligned} \frac{M}{M_0} = {} & \frac{2}{3}\frac{E\varepsilon_{\max}}{\sigma_c}\left(\frac{\bar{y}}{c}\right)^3 + \left(1 - \left(\frac{y}{c}\right)^2\right) \\ & + \frac{\sigma_c}{\sigma^y}\frac{1}{a^2}\left[(1-a^2x^2)\ln(1+ax) + (1-ax)^2 - (1-a^2)\ln(1+a) - (1-a)^2\right] \\ & + \frac{F}{3\sigma^y}\frac{(2\varepsilon_{\max}+\varepsilon_w)}{\varepsilon_{\max}^2}\langle \varepsilon_{\max} - \varepsilon_w \rangle^2 \end{aligned}$$

where $M_0 = bc^2\sigma^y$ is the fully plastic static moment, $a = E\dot{\varepsilon}_{\max}\tau/\sigma_c$ and $x = y/c$.

The above expression has been worked out for three strain rates, 0·01, 0·15 and 12 $\sec^{-1}$ and the resulting moment curves are shown in Fig. 3. Also shown are experimental curves at strain rates of 0·15 and 12 $\sec^{-1}$ carried out by Aspden [8]. The material of the beams tested by Aspden was a mild steel of slightly different composition and preparation than that from which the constants were obtained. There is qualitative agreement in that the very high upper yield moment and rapid relaxation to the lower yield moment which are a notable feature of Aspden's results are predicted.

4. TRANSVERSE IMPACT OF A MASS ON A BEAM

In this section the upper yield point theory will be used to predict the onset of yielding in an infinite elastic beam of cross sectional area S, moment of inertia I, density ρ and modulus of elasticity E that is struck transversely by a mass m moving before the instant of impact with velocity V, and which remains in contact with the beam after impact. It can be shown [9] that the bending moment at the point of impact in this case is given by

$$M(0,t) = 2EIK^2V\,e^{\alpha^2 t}\operatorname{erfc}[\alpha(\sqrt{t})] \tag{13}$$

where $K = (\rho S/4EI)^{\frac{1}{4}}$ and $\alpha = 8EIK^3/m$. The above equation implies that the bending moment at the instant of impact jumps to the value $2EIK^2V$, which is independent of m

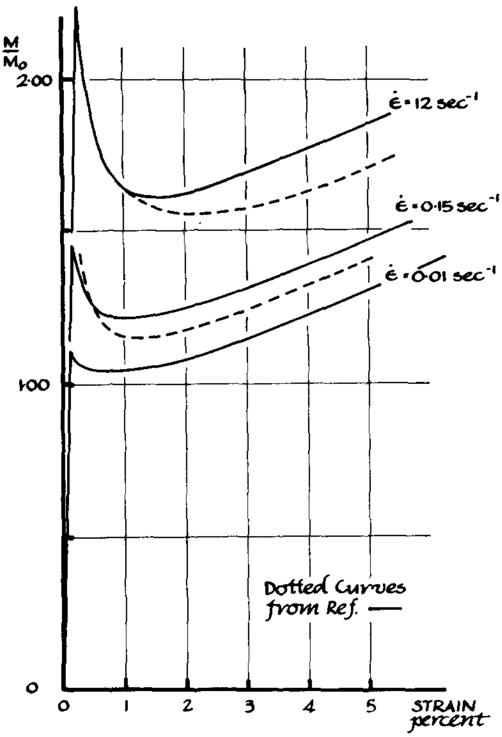

FIG. 3. Moment curves at constant rate of curvature. Dotted curves are experimental results by Aspden [8].

and decays from this value more or less rapidly depending on α and thus on m. If the upper yield point is ignored then the velocity to produce plastic behavior is given by

$$V/C = (r/y)\varepsilon_y$$

where $r = (I/S)^{\frac{1}{2}}$ is the radius of gyration of the section, ε_y is the yield strain and C the velocity of longitudinal waves in the beam material. However, if the upper yield point is considered the possibility exists that for finite values of the mass the stresses induced by the impact might decay rapidly enough that they fall to values below the yield stress in a time which is less than the delay time at the most highly stressed fiber.

The delay time t_d for the extreme fiber is given by (10) with (13) in the form

$$\frac{d^3}{A}\left(\frac{2EIK^2Vy}{\sigma_0 I}\right)^{\beta}\int_0^{t_d} e^{\beta\alpha^2 t}\{\operatorname{erfc}[\alpha(\sqrt{t})]\}^{\beta}\,dt = 1.$$

Due to the high value of the exponent β the contribution to the integral when the stress is below the lower yield point is negligible and thus to obtain a bound on V for finite m it is enough to set $t_d \to \infty$. Using the substitution $s = \alpha^2 t$ and suitable manipulations the above equation can be transformed into an inequality for the range of velocity leading to elastic impacts. Using the notation

$$\lambda(\beta) = \left[\int_0^{\infty} e^{\beta s}[\operatorname{erfc}(\sqrt{s})]^{\beta}\,ds\right]^{-1/\beta}$$

the result becomes

$$\frac{V}{C} \leq \frac{r\sigma_0}{yE}\left(\frac{A}{d^3}\right)^{1/\beta}\left(\frac{m}{8EIK^3}\right)^{-2/\beta}\lambda(\beta).$$

The integral for $\lambda(\beta)$ is easily computed numerically and on substituting the same values of the parameters as used in Sections 2 and 3 the limiting velocities for the two grain sizes are given by

$$\frac{V}{C}\frac{y}{r} \leq 1{\cdot}75\times 10^{-3}\left(\frac{m}{8EIK^3}\right)^{-2/9} \qquad \text{for 2030 grains/mm}^2$$

and

$$\frac{V}{C}\frac{y}{r} \leq 1{\cdot}295\times 10^{-3}\left(\frac{m}{8EIK^3}\right)^{-2/9} \qquad \text{for 346 grains/mm}^2$$

These results are shown in Fig. 4 from which it is clear that for values of $m/8EIK^3$ greater than 1 the increase in V predicted by this theory over that predicted by the simple theory is not important. However, for smaller values of $m/8EIK^3$ the increase in V can be very substantial. The application of this to the design of highway guard-rails is being explored. In this section y is used to denote the distance of the outermost fiber from the neutral axis.

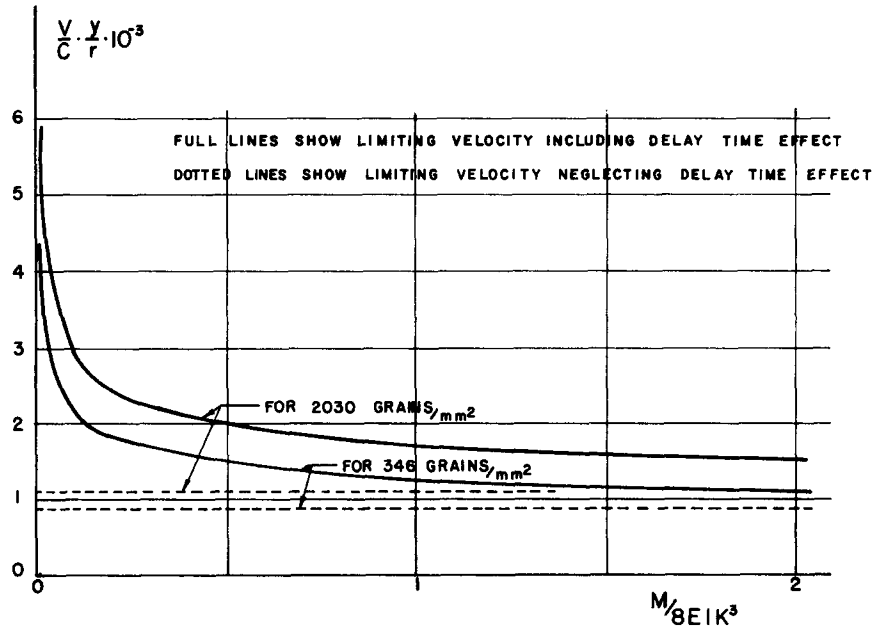

FIG. 4. Limiting velocity for elastic impact by finite mass.

5. DISCUSSION

It has been the purpose of this paper to demonstrate the possibility of predicting the behavior of a beam of a material prescribed by a constitutive equation and to relate this to experimental results. Although the agreement between the prediction and the experiment is not perfect (the differences are mainly in the strain hardening region and may be due to differences in the structures of the materials considered here and used for the beam tests), it is in agreement with the general features of the actual response. The surprising features of the experimental results namely, the very high upper yield moment and the rapid relaxation to the lower yield moment are present in the solutions.

At the present time the accuracy of much of the experimental work on dynamic material behavior is not adequate to allow its use directly in applications. The scatter of the experimental results is such as to call for smoothing technique, by eye for example or by a least squares method. Whether either of these methods lead to the best result cannot be ascertained. However, a procedure such as that used to obtain σ_c and τ subjects the data to a special type of averaging which as we have shown allows useful predictions to be made of the response of the material in more complex mechanical situations within the same range of strain and strain rate. The possibility of using much of the available experimental data in structural problems is a very important one since the strain rates necessary to produce a doubling of the yield moment are of the order of 10 $\sec^{-1}$ which could be experienced by building structures during earthquakes or other forms of impact loading. The problem of impact of a mass on a beam examined in Section 4 although it only touches the surface of this field shows that the consequences of rate effects in structural problems can be substantial.

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REFERENCES

[1] N. J. HUFFINGTON (ed.), *Colloquium on the Behavior of Materials under Dynamic Loading*. American Society of Mechanical Engineers, New York (1965).
[2] L. E. MALVERN, *J. appl. Mech.* **18**, 203 (1951).
[3] S. RAJNAK and F. HAUSER, Report No. UCRL-10638, Lawrence Radiation Laboratory, University of California, Berkeley (1963).
[4] J. D. CAMPBELL and K. J. MARSH, *Phil. Mag.* **7**, 933 (1962).
[5] V. V. SOKOLOVSKY, *Prikl. Mat. Mekh.* **12**, 261 (1948).
[6] C. RIPARBELLI, *Proc. Soc. exp. Stress Analysis* **14**, 55 (1954).
[7] K. J. MARSH and J. D. CAMPBELL, *J. Mech. Phys. Solids* **11**, 49 (1963).
[8] R. J. ASPDEN, D.Phil. Thesis, Oxford University Engineering Laboratory (1963).
[9] J. M. KELLY, *Int. J. Solids Struct.* **3**, 191 (1967).

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Résumé—Une description empirique de la proportion du comportement sensible de l'acier doux basée sur une dépendance logarithmique de l'effort d'écoulement sur la proportion de tension est examinée en détails. Il est démontré que la loi pourvoir une bonne représentation de reaction de matériel pour des tensions allant jusqu'à 5.0%. Le phénomène de rendement délayé, qui est une caractéristique frappante du comportement de l'acier doux est inclu et un mécanisme, antérieurement trouvé exact dans la représentation du délai de temps sous des historiques de tensions complexes. La relaxation du point supérieur de rendement résultant du délai de temps en une situation de proportion de contrainte constant est prescrite par la proportion de comportement sensible du matériel.

Il est démontré que la réaction du matériel est fonctionnelle à l'historique de la contrainte et a une faible mémoire en tension aussi bien qu'en temps. La tension caractéristique de mémoire est constatée petite ainsi pour de grandes tensions l'influence de l'historique est petite et une équation de représentation d'état est appropriée. Pour de petites tensions et des historiques de contrainte pour les types changeant rapidement la représentation intégrale doit être employée.

La théorie est appliquée à la prédiction de la réaction d'une poutre sujette à une proportion constante de courbure et la réaction prédite comparée avec des données expérimentales publiées. De plus, Le choc transversal d'une masse sur une poutre est investigué et la théorie est employée pour prévoir les charges de rendement sur la poutre. C'est un important problème pour les plans des rails de protection des autoroutes.

Zusammenfassung—Eine empirische Beschreibung des geschwindigkeitsabhängigen Verhaltens von Stahl, das auf der logarithmischen Abhängigkeit des Spannungsflusses von der Dehnungsgeschwindigkeit beruht wird gegeben und genau untersucht. Es wird gezeigt, dass die Regel eine gute Darstellung für Spannungen bis zu 5% gibt. Das Phenomen des verzögerten Flusses, eine bemerkenswerte Eigenschaft unlegierten Stahles ist inbegriffen, wie auch der Mechanismus der früher gefunden wurde und die Verzögerung genau darstellt, auch wenn die Umstände komplex sind. Die Relaxation von der oberen Fliessgrenze, als Resultat der Verzögerung bei konstanter Spannung, wird als Eigenschaft des Geschwindigkeitsabhängigen Materiales beschrieben. Es wird gezeigt, dass das Ansprechen des Materiales eine Funktion der Spannunggeschichte ist, das Gedächtnis wird geschwächt sowohl in Zeit wie in Spannung. Die charakteristische Gedächtnisspannung ist so klein, dass der Einfluss des Schwindens bei grossen Spannungen unbedeutend ist, und eine Gleichung die den Zustand gibt ist angebracht. Für kleine Spannungen und Geschichten die sich schnell ändern, muss Integral-Darstellung angewandt werden.

Die Theorie wird angewandt um das Verhalten eines Balkens vorherzusagen, der ständiger Biegung unterliegt, die Vorhersage wird mit veröffentlichten Versuchsresultaten verglichen. Ferner wird der Querschlag einer Masse auf einen Balken untersucht und die Theorie angewandt um das Fliessen vorherzusagen. Das Problem ist für Strassen-Schutzgelände von Wichtigkeit.

Абстракт—Исследуется в деталях эмпирическое описание поведения мягкой стали, чувствительной к скорости, основанного на логарифмической зависимости течения напряжения от скорости деформации. Показано, что закон предоставляет хорошее изображение ответной реакции материала для деформаций до около 5.0%. Включено явление задержания текучести, которое поразительно характерно для поведения малоуглеродистой стали и ранее найденный механизм точен в изображении истории времени задержки под сложным напряжением. Релаксация от верхней точки текучести, получающаяся в результате времени задержки в положении постоянной скорости деформации приписывается поведению чувствительного к скорости материала.

Указывается, что ответная реакция материала представляет из себя функцию истории напряжения и обладает затухающей памятью, как деформации, так и времени. Характерная деформация памяти найдена малой так что для больших деформаций влияние истории не велико и уравнение представления состояния соответственно. Для малых деформаций и историй деформации быстро изменяющегося типа должно употребляться интегральное изображение.

Для предсказания ответной реакции балки применяется теория в зависимости от постоянной скорости кривизны и предсказанная ответная реакция сравнивается с опубликованными экспериментальными данными.

В дополнение исследуется поперечный удар массы на балку и применяется теория, чтобы предсказать начало пластической деформации в балке.

Проблема важна при проэктировке дорожного ограждения.